Application of Linear-Phase Digital Crossover Filters to Pair-Wise Symmetric Multi-Way Loudspeakers Part 2: Control of Beamwidth and Polar Shape

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ABSTRACT

In part 2, we present an alternate simplified design technique that is based not on Part 1's specification of frequency responses at arbitrary off-axis vertical angles, but on specification of the total shape and coverage angle (vertical beamwidth) of the polar patterns generated by pairs of separated point sources. Here we show that when only a single pair of drivers is operating at a specific frequency (Part 1 called these the "critical frequencies"), the spacing of these drivers is a constant distant apart in terms of acoustic wavelength. The design procedure of Part 2 simplifies that of part 1 by restricting the level of the forced-to-be-flat off-axis angle to -6 dB thus making it equal to the level of the polar beamwidth specification, i.e. beamwidth is defined as the angle between the 6-dB-down points from on axis. Thus restricted, Part 2 shows that the spacing of each pair of drivers at their critical frequencies should be in the range of 0.4 to 0.6 wavelength to yield well-behaved polar shapes with beamwidths in the range of 67º to 113º. Part 2 also shows that that spacing ratios between successive pairs of drivers should preferably be in the range of 2:1 to 2.5:1, but can extend out to 4:1, but at the expense of polar uniformity at angles beyond the 6-dB-down points.

0 INTRODUCTION

Part 2 of this paper describes an alternate analysis and design procedure for the DSP linear-phase loudspeaker crossover filter technique described in Part 1. This alternate analysis and design procedure stresses performance parameters important to pro-sound applications such as beamwidth, directivity, and polar-pattern shape, rather than parameters typically stressed in domestic speaker design applications where maintenance of uniform off-axis frequency response is emphasized. The alternate design technique simplifies the array design for both domestic and professional applications.

Part 2 first presents a quick review and analysis of the design technique of Part 1 in the introduction and Section 1. It then presents a new set of crossover frequency-response design equations that result in a system that provides constant beamwidth in Section 2. And then closes with a summary of the design procedure in Section 3 along with the design of an example constant-coverage/directivity array loudspeaker system followed by a detailed simulation of the array's performance in Section 4.

Part 1 describes a crossover design technique for multi-way loudspeaker systems that provides reasonably-flat and smooth frequency responses, not only at a single point in space, but within a wide region in front of the system. The crossover technique is intended to operate with multi-way systems with drivers arranged pair-wise symmetrical around a central tweeter. The design method is based on specifying a crossover frequency-response shape that forces a flat frequency response at a specified vertical off-axis angle. When thus specified, frequency responses at other vertical off-axis angles are found to be reasonable flat as well.

The technique is distinctively different and provides superior polar performance to other crossover techniques loosely classified as Linkwitz-Riley filters, constant-voltage filters, and DiAppolito configurations. The technique is also loosely related to so-called "log" arrays, but is superior because designs can be implemented with fewer drivers that can be spaced at un-equal ratios. The uniquely-shaped zero-phase crossover frequency responses that result from his design method require the use of DSP techniques for implementation and can't be easily implemented with analog filters.

This report presents an alternate look at the new crossover technique based not on specification of frequency responses at certain off-axis vertical angles, but on specification of the total shape and coverage angle (vertical beamwidth) of the polar patterns generated by pairs of separated point sources.

What is unique about the crossover technique of Part 1 is that at any specific frequency, only a single pair of drivers or two pairs of drivers are operating simultaneously. This feature allows the design technique to apply both to equal and unequal-spaced pairs of drivers. Spacing in this sense applies to the spacing of one pair of drivers with respect to the next pair of drivers.

To cover a broad frequency range, the symmetric spacing of the drivers has to progressively increase going from the tweeter outward. Spacing can either be equal percentage, i.e., 1 : 2 : 4 : 8 or unequal, i.e. 1 : 2.2 : 5.5 : 13.2. It is noted in passing that the "log" array designs [5, Part 1] almost always specify equal-percentage spacing of the drivers.

When only a single pair of drivers is operating at a specific frequency, the spacing of these drivers is a constant distant apart in terms of acoustic wavelength. The frequencies at which single pairs of drivers operate independently are called "critical" frequencies.

This report points out that the symmetrically-placed pairs of drivers that compose the array are all spaced a constant distant apart in terms of acoustic wavelength at the critical frequency of each of the crossover's bands. This is the heart of the array's capability to maintain flat frequency response at various vertical off-axis angles and to maintain constant vertical coverage and directivity over a broad frequency range.

Although the individual pairs of drivers are spaced a constant wavelength apart only at a discrete set of crossover band critical frequencies, the array's constant-coverage behavior is maintained between these frequencies by the unique shape of the crossover's frequency responses driving adjacent pairs of drivers. Part 1 shows that a linear combination of the acoustic outputs of adjacent pairs of drivers can maintain essentially constant frontal coverage not only at the band's critical frequencies but at all frequencies in between.

This report shows that this technique of linearly summing the acoustic outputs of adjacent pairs of drivers not only yields flat vertical off-axis frequency responses, but approximately maintains the overall shape of the vertical polar patterns at frequencies between the band's critical frequencies. In other words, the polar pattern shapes at intermediate frequencies between the band critical frequencies are very close to the shape of the polar pattern that is generated by a single pair of sources separated by the specified wavelength distance. The maintenance of constant polar shape with frequency means that the system will maintain constant coverage and directivity with frequency, and provide uniformly-flat off-axis frequency responses over a wide range of off-axis angles.

The analysis in Part 2 suggests that the spacing of each pair of drivers at their critical frequencies should be in the range of 0.4 to 0.6 wavelength. This spacing will yield desirable polar shapes with relatively narrow main beams and side lobes that are at least 10 dB down. Analysis shows that a critical spacing of one-half wavelength is near optimum and provides a polar pattern with no side lobes and a vertical beamwidth of about 84º. Other spacings in the suggested 0.4 to 0.6 wavelength range will provide vertical beamwidths in the range of 67º to 113º.

This report also shows that spacing ratios between successive pairs of drivers should be in the range of 2:1 to 2.5:1 to maintain polar uniformity out to a level of about 12 to 15 dB down from on axis. Spacing ratios in the range of 2.5:1 to 4:1 can be implemented, but at the expense of polar uniformity at angles beyond the 6-dB-down points.

1 REVIEW OF THE DESIGN PROCEDURE OF PART 1

The crossover method described in Part 1 applies to a loudspeaker array configuration composed of a single central tweeter surrounded symmetrically by pairs of lower-operating-frequency transducers arranged in a vertical line. Figure 1 left in Part 1 shows an array configuration for an example three-way design.

After specifying the configuration of the array, i.e. the driver types, sizes, and their locations, the design task described in Part 1 is to calculate the frequency responses of the crossover filters that drive each pair of loudspeakers and the central tweeter. The design method for computing these responses is based on three steps:

1. Specify a vertical off-axis attenuation factor for the pressure level generated by the array at a specific off-axis angle (Effectively this sets the desired vertical coverage of the array by specifying that the frequency response will be flat at the specified vertical off-axis angle at the specified attenuation, i.e. specifying an attenuation factor of 0.25 at 45º forces the ±45º vertical off-axis frequency responses to be flat at a level of -12 dB down from on axis.),

2. Calculate the crossover band critical frequency for each pair of speakers. These frequencies are actually the approximate centers of each of the operating bands and at these frequencies, only a single pair of speakers are operating, and

3. Calculate the actual frequency responses for each of the filters that drive each pair of drivers and separately for the tweeter.

The crossover frequency responses are very distinctive because each band's frequency response exhibits a sharp point on the top with the response falling rapidly on either side. The following figure (Fig. 1) illustrates this behavior for an example six-way system. The pointed tops coincide with the crossover band critical frequencies. Each band's frequency response essentially falls to infinite attenuation at frequencies above and below the adjacent band's critical frequencies. Each pair of drivers operates only in a very restrictive frequency band bracketed by the adjacent pair's critical frequencies.

The traditionally defined crossover frequencies occur at the 6-dB-down points on the curves where the responses overlap. The lowest and highest frequency drivers (woofers and tweeter) cover wider frequency bands. Because the tweeter operates by itself above a certain frequency, off-axis frequency response cannot be controlled by the crossover and must be controlled by the polar characteristics of the tweeter itself.

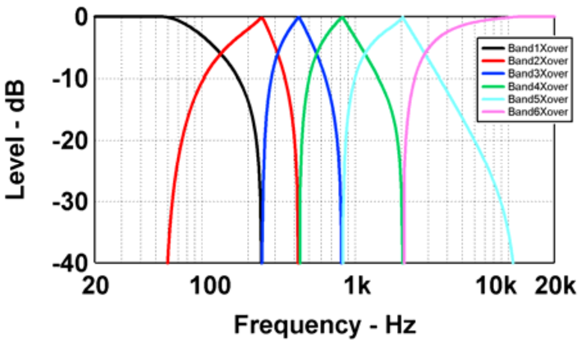


Fig. 1. Crossover frequency responses for a six-way speaker system following the design technique of part 1. Each band's crossover only drives a single pair of speakers. At any specific frequency, only a single pair of drivers or at most two pairs of drivers are operating. At the pointed peak of each crossover band, called a critical frequency, only one pair of drivers are operating. This crossover has critical frequencies of [56, 247, 430, 824, and 2060] Hz.

The crossover filtering technique is quite unique because at any specific frequency, only one pair or at most two pairs of speakers are operating simultaneously (the single center tweeter is the sole exception which operates by itself at high frequencies).

2 AN ALTERNATE VIEWPOINT AND DESIGN TECHNIQUE

The following describes another way of looking at the crossover design technique of Part 1 from the standpoint of polar shape.

2.1 Constant Wavelength Spacing

Analysis reveals that at the critical frequency of each crossover band (the pointed tops of the frequency responses), the center-to-center spacing of each of the pairs of drivers is a constant fraction of the wavelength at each of the corresponding critical frequencies. It is this constant acoustic size, with respect to wavelength, that is the basis for the arrays wide-band constant-coverage behavior.

This suggests an alternate way to compute the band critical frequencies:

$$f_C = \frac{R_\lambda c}{D} = \frac{R_\lambda c}{2X} \qquad (1)$$

where

f_C = Band critical frequency

R_λ = wavelength ratio ($= \frac{D}{\lambda}$)

D = Center-to-center spacing of sources

c = speed of sound

X = driver spacing from center ($= D/2$)

2.2 Polar Pattern Created by Two Separated Point Sources

The next graph (Fig. 2) shows various frontal polar patterns for a pair of vertically-oriented point sources separated by a specific distance in wavelengths. The polar patterns only show the front-facing polar. The polar pattern is front-rear symmetric and thus the rear pattern is not shown. The -6 dB beamwidth is also indicated under each figure. For separations greater than one-half wavelength (Fig. 2 d - f), the level of the side lobes are also indicated.

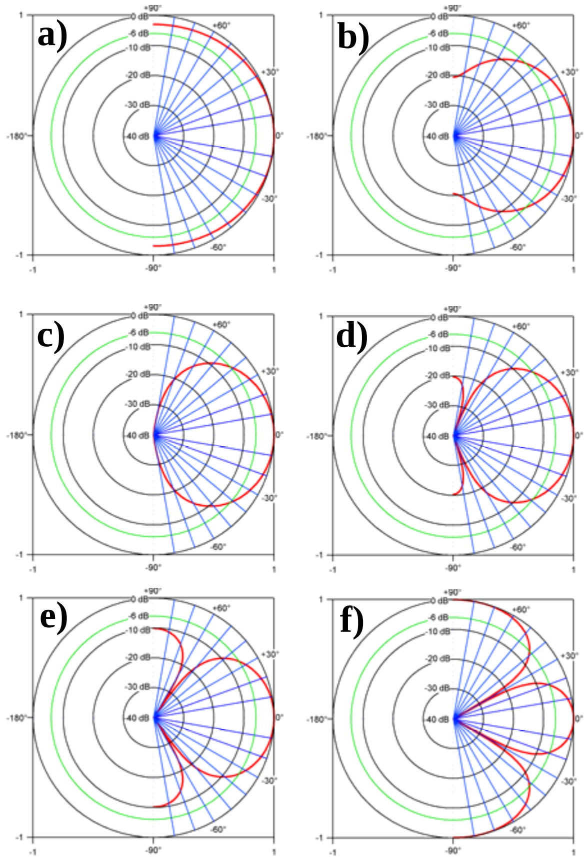


Fig. 2. Various vertical-plane frontal polar plots for a pair of vertically stacked point sources. The parameter D/λ is the spacing to wavelength ratio. a) D/λ = 0.1, beamwidth = 180º, b) D/λ = 0.4692, beamwidth = 90º, c) D/λ = 0.5, beamwidth = 83.6º (here, the points are separated by one-half wavelength and produce a perfect polar with no side lobes), d) D/λ = 0.53, beamwidth = 77.5º, side lobe down 20 dB, e) D/λ = 0.6, beamwidth = 67.1º, side lobe down 10 dB, f) D/λ = 1, beamwidth = 38.7º, side lobe down 0 dB.

The following two figures (Figs. 3 and 4) show how the beamwidth (-6 dB) angle and side-lobe level varies with source separation.

The vertical beamwidth of two vertically-oriented point sources $\theta_{\deg s}$ as a function of the spacing-to-wavelength ratio R_λ can be shown as:

$$\theta_{\deg s} = \frac{360}{\pi}\arcsin\left(\frac{1}{3R_\lambda}\right) \quad (2)$$

This in turn may be solved for the wavelength ratio or critical wavelength yielding:

$$R_\lambda = \frac{D}{\lambda} = \frac{1}{3\sin\left(\frac{\pi\theta_{\deg s}}{360}\right)} \quad (3)$$

Figure 3, calculated using Eq. 2, shows that for small separations of less than a third of a wavelength, the vertical coverage is essentially omni directional with a coverage angle of 180º. Above a separation of 1/3rd wavelength, the beamwidth starts to decrease and then falls monotonically to about 39º at a spacing of one wavelength. Indicated on the graph is the approximate effective source separation range that minimizes side lobes. Separations in the range of 0.4 to 0.6 wavelength hold side lobes to levels no more than 10 dB down from the on-axis level.

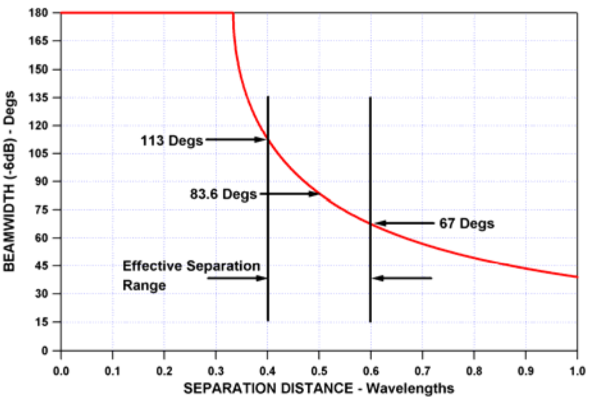


Fig. 3. Vertical beamwidth (-6 dB) vs. separation distance for two vertically-aligned point sources calculated using Eq. 2. The beamwidth remains omni-directional (180º) up to a separation distance of 1/3rd wavelength (D = 0.333). For design purposes, effective source separations between 0.4 to 0.6 wavelength yield side lobes that are -10 dB or lower. (see next figure). In this range, beamwidths in the range of 67º to 113º can be selected. A separation distance of one-half wavelength (D = 0.5 λ) provides a beamwidth of about 84º with no side lobes (middle of graph, see Fig. 2e for polar).

The side lobe level (at ±90º) L_{90} as a function of the source spacing to wavelength ratio R_λ can be shown to be:

$$L_{90} = \left|\cos\left(\pi R_\lambda\right)\right| \quad (4)$$

Figure 4, computed from Eq. 4, shows how the side-lobe level (at ±90º) varies as a function of the source spacing. A source spacing of one-half wavelength results in no side lobes. As in the previous figure, the approximate effective source separation range that minimizes side lobes is also shown on the graph. For design purposes, separations in the range of 0.4 to 0.6 wavelength hold side lobes to levels no more than 10 dB down from the on-axis level.

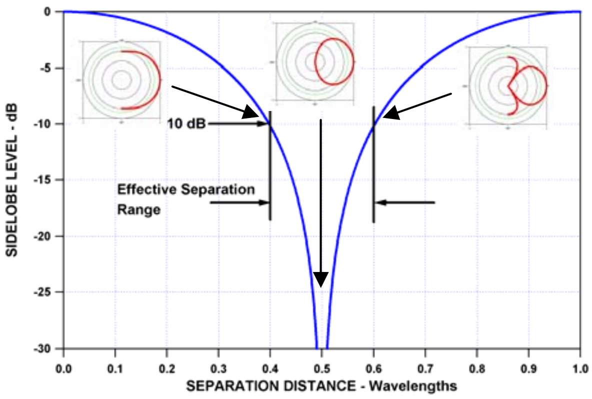


Fig. 4. Side lobe level (at ±90º) vs. separation distance for two point sources. For design purposes, effective source separations between 0.4 to 0.6 wavelength yield side lobes that are 10 dB down or lower. A separation distance of one-half wavelength (D = 0.5 λ) results in no side lobes. Example mini polar patterns are shown at the extremes and center of the suggested design range.

2.3 Linear Combination of the Patterns of Two Pairs of Separated Point Sources

Part 1 shows that a linear combination of the acoustic outputs of adjacent pairs of drivers can maintain essentially flat off-axis frequency response over a significant range of off-axis vertical angles. Part 2 will show that this linear combination of outputs also approximately maintains the vertical polar shape in the same range of frequencies.

Part 1's design technique depends on the characteristic that the overall polar pattern of two pairs of separated drivers at a specific frequency can essentially be made equal to the polar pattern of a single pair of drivers at a specific wavelength spacing by linearly combining the outputs of the two adjacent pairs of drivers. In other words, even though the polar patterns of each of the two pairs of drivers considered individually at a specific frequency do not match the desired pattern, a linear combination of the two dissimilar patterns can match the desired pattern.

This section illustrates this behavior and shows how the polar pattern of one pair of point sources may be linearly combined with the polar pattern of another pair of point sources separated by another distance to yield a desired pattern.

To illustrate the summing behavior, consider the following configuration of four point sources with a normalized spacing of [-X2, -X1, +X1, +X2] = [-1.0, -0.5, +0.5, +1.0] and driven in pairs:

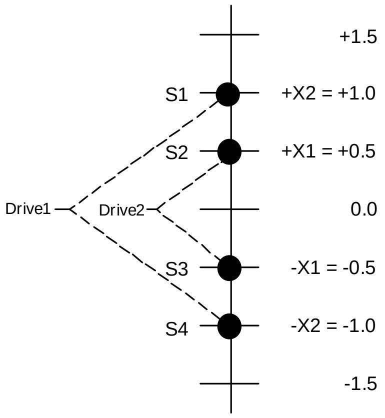


Fig. 5. Configuration of four point sources (S1 - S4) oriented vertically at normalized locations of -1.0, -0.5, +0.5, and +1.0 and driven in pairs: Drive1 to S1 and S4, and Drive2 to S2 and S3. At the lower critical normalized frequency of 0.285, the outermost sources S1 and S4 are separated by 0.57 wavelength. Correspondingly, at a the higher critical frequency which is twice as high, the innermost sources S2 and S3 are also separated by 0.57 wavelength.

At a normalized frequency of 0.285 (normalization essentially is the same thing as assuming a unit speed of sound, i.e. c = 1 unit/sec)), the separation of the [-X2, +X2] = [-1.0, +1.0] pair is 0.57 wavelength and exhibits the polar pattern shown previously in Fig. 2c. Likewise, at a normalized frequency of 0.57 (twice as high), the separation of the [-X1, +X1] = [-0.5, +0.5] pair is also 0.57 wavelength and exhibits the same polar pattern. These two frequencies are the critical frequencies for these pair of sources.

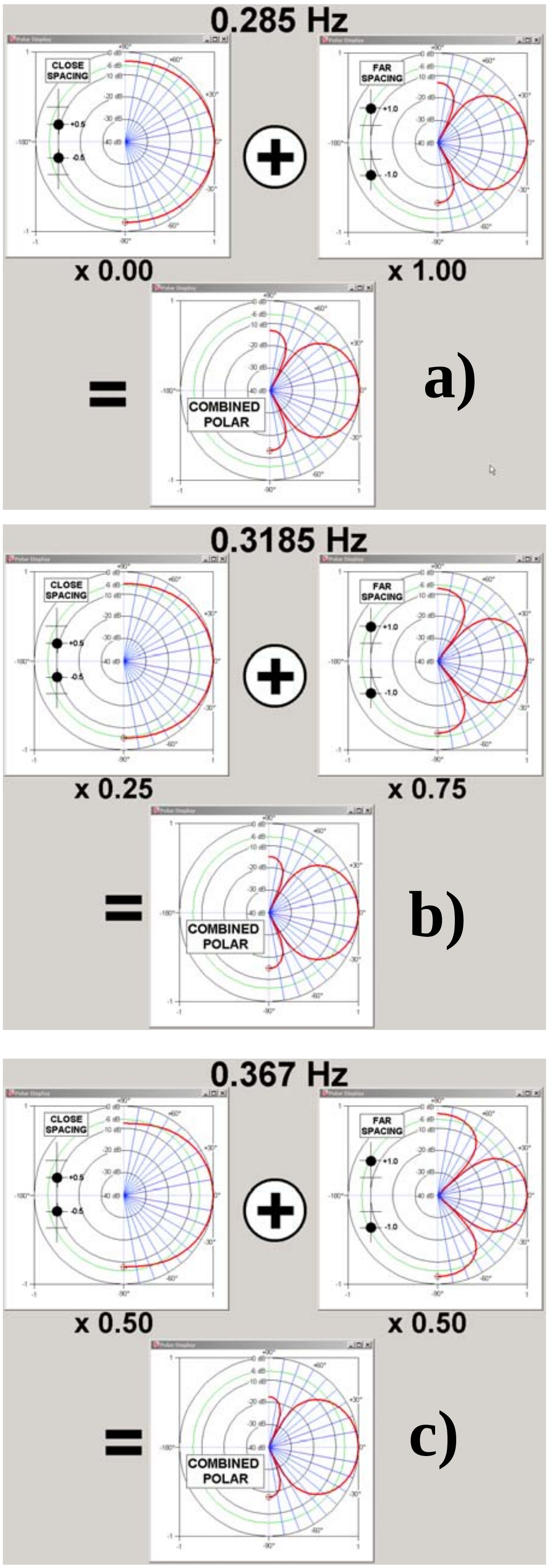
However, at intermediate frequencies between the critical frequencies of 0.285 and 0.57, each pair of sources considered individually exhibits a polar pattern which differs significantly from the desired pattern of Fig. 2c. In spite of this, the following figure will show that the desired polar pattern may be approximated by a linear sum of the patterns of each pair of sources.

Effectively, at any intermediate frequency, you can vary the relative drive levels of each pair of sources to create nearly the same polar pattern as provided by a single pair at the frequency extreme or critical frequencies. When the acoustic radiation from each pair of sources is coherently summed, the resultant total radiation will have the correct desired pattern even though the patterns of the individual pairs of sources are incorrect.

The polar summing operation is illustrated in the following figure (Fig. 6) for the configuration shown in Fig. 5 with individual sets of polar patterns at five frequencies. The frequencies and corresponding summing ratios are shown in the following Table 1. The frequencies include the two end or critical frequencies (frequencies 1 and 5) and three intermediate frequencies (2 to 4) where weighted sums of the individual patterns approximately add up to the patterns at the end frequencies. The middle frequency (3), where the drive ratios are equal, is the crossover frequency between the two pairs of sources. This design generates a constant beamwidth of 71.5º for two pairs of sources spaced in a 2 to 1 ratio over an octave frequency range between critical frequencies where the sources are spaced at 0.57 wavelength apart.

Table 1: Frequency and Summing Ratios

Frequency Identifier	Normalized Frequency	Drive1 (Outermost Sources S_1 and S_4)	Drive2 (Innermost Sources S_2 and S_3)
1) Lower critical frequency	0.285	0.00 (Off)	1.00 (Full on)
2) Intermediate frequency	0.3185	0.25	0.75
3) Intermediate frequency (crossover)	0.367	0.50	0.50
4) Intermediate frequency	0.443	0.75	0.25
5) Upper critical frequency	0.570	1.00 (Full on)	0.00 (Off)



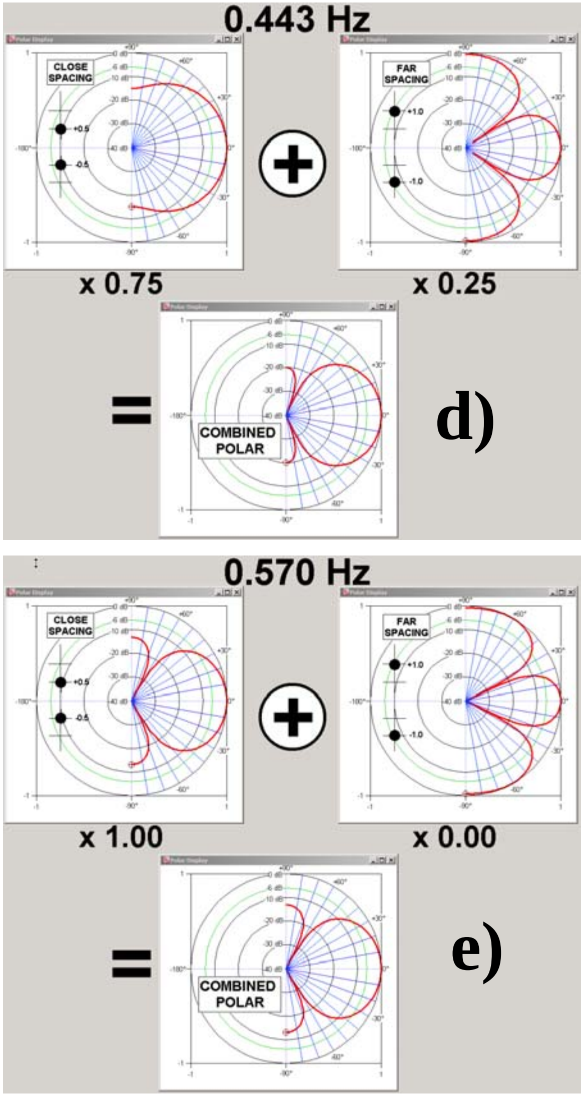


Fig. 6. Illustration of polar pattern summing for the configuration of Fig. 5 at normalized frequencies of a) 0.285, b) 0.3185, c) 0.367, d) 0.443, e) 0.570. These series of graphs illustrate that it is possible to approximate a desired polar pattern by linearly summing the patterns of two pairs of sources over a range of frequencies, although the patterns of the individual pairs of sources may be incorrect.

Note that all the combined polars in these five figures (bottom polar in each figure) have very uniform main lobes, but exhibit greater variations in the side lobe. The combining operation was chosen to maintain the same 6-dB-down beamwidth at all frequencies (71.5º here), i.e. the level at ±35.75º (=71.5º/2) off the main axis is exactly -6dB. The level at greater off-axis angles has larger variability.

2.4 Variation of Off-Axis Polar Shape with Different Spacing Ratios

In this section an investigation is made of the effect of the spacing ratio of the two pairs of sources on the total polar response shape. This was done by generating a set of overlaid polar responses at several spacing ratios in the range of 2:1 to 4:1. At each spacing ratio, a polar overlay was generated by varying the frequency in ten steps between the two critical frequencies so as to maintain a constant beamwidth. The frequencies correspond to summing ratios from 0 to 1 in steps of 0.1. Two sets of overlays were created at critical wave length separations of 0.5 and 0.57. These separations correspond to beamwidths of 83.5º and 71.2º. The overlays for these two cases are shown in Figs. 7 and 8.

The following figure (Fig. 7) shows pattern overlays for a critical wavelength spacing of 0.5 at four different spacing ratios of (a) 2:1, (b) 2.5:1, (c) 3:1, and (d) 4:1. Note that the polar shapes are all nearly identical between the included 6-dB-down beamwidth angles. However, as the spacing ratio increases, the polar uniformity at angles beyond the 6-dB-down angle decreases dramatically. For best polar uniformity between the critical frequencies, spacing ratios between 2:1 (a) and 2.5:1 (b) appear to offer the best results.

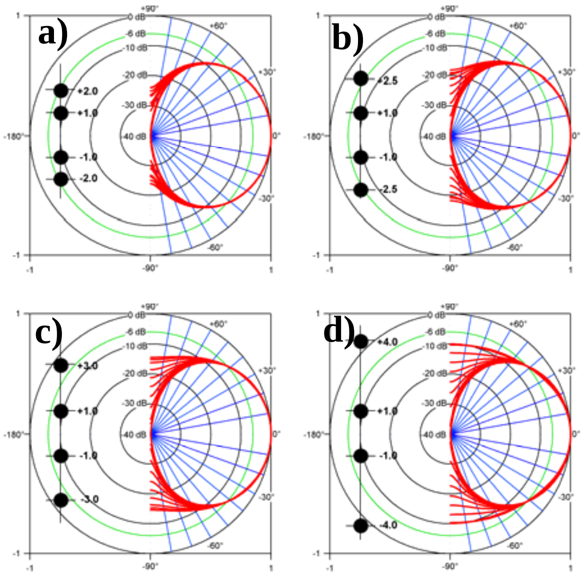


Fig. 7. Polar pattern overlays for pairs of sources separated by different spacing ratios for a critical design wavelength of 0.5 λ: a) ratio = 2:1, b) ratio = 2.5:1, c) ratio = 3:1, and d) ratio = 4:1. To generate each polar overlay, the frequency was varied in 10 steps between the two critical frequencies so as to maintain a constant beamwidth of 83.6º (see Fig. 2c). The frequencies corresponded to summing ratios from 0 to 1 in steps of 0.1. Note that as the spacing ratio increases, the polar uniformity at angles beyond the 6-dB-down angle decreases dramatically. Spacing ratios in the range of 2:1 to 2.5:1 [(a) and (b)] provide good uniformity out to about 12 to 15 dB down from on axis.

The following figure illustrates the polar pattern overlays for a critical wavelength spacing of 0.57 at two different spacing ratios of (a) 2:1, and (d) 4:1. This particular wavelength spacing exhibits a beamwidth of 71.2º and a shape which exhibits a 13-dB-down lobe at ±90º (not shown in Fig. 2). As with the previous set of overlays (Fig. 7), the best main lobe uniformity is exhibited by the lower 2:1 spacing ratio (a).

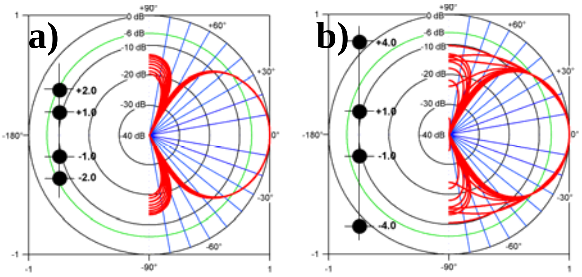


Fig. 8. Polar pattern overlays for pairs of sources separated by different spacing ratios for a critical design wavelength of 0.57 λ: a) ratio = 2:1, and b) ratio = 4:1. To generate each polar overlay, the frequency was varied in 10 steps between the two critical frequencies so as to maintain a constant beamwidth of 71.2º. Although this critical design wavelength has a lobe at ±90º. The main lobe is still much more uniform for the smaller spacing ratio.

2.5 Calculation of Crossover Frequency Responses

2.5.1 *Response Between Two Pairs of Sources*

The following two equations give the constant-beamwidth crossover frequency responses for a symmetrical dual pair of sources whose configuration was shown previously in Fig. 5. Equation 5 gives the low-pass frequency response for the wider-spaced (outside) pair of sources. Equation 6 gives the high-pass frequency response for the narrower-spaced (inside) pair of sources. They are derived from Eq. 9 in part 1 by setting *a*=0.5. The equations are frequency normalized to span only the range between the two critical frequencies of the source pairs, i.e. $1 \le f_N \le R$ where $f_N = f / f_{C1}$ and R is the ratio between the two critical frequencies ($R = f_{C2} / f_{C1}$) or equivalently the spacing ratio of the dual pair of sources (X_2/X_1).

$$H_{LP}(f_N, R) = \begin{cases} \dfrac{1}{2}\left| \dfrac{2\cos\left(\dfrac{\pi f_N}{3R}\right) - 1}{\cos\left(\dfrac{\pi f_N}{3}\right) - \cos\left(\dfrac{\pi f_N}{3R}\right)} \right| & \text{for } 1 \le f_N \le R \\ 0 & \text{otherwise} \end{cases} \qquad (5)$$

Where:

H_{LP} = Low-pass filter function

f_N = Normalized frequency ($= f / f_{C1}$)

R = Ratio between the critical frequencies ($= f_{C2} / f_{C1} = X_2 / X_1$) or the spacing ratio between the two pairs of sources

f_{C1} = Lower critical frequency (frequency where the spacing of the farthest separated pair of sources is a specific wavelength)

f_{C2} = Upper critical frequency (frequency where the spacing of the closest separated pair of sources is the same specific wavelength)

X_2, X_1 = Source coordinates (X_1 is the coordinate of the closest separated pair of sources and X_2 is the coordinate of the farthest separated pair of sources)

Implicit in the derivation of this equation is that the beamwidth at the 6-dB-down points is maintained at all frequencies, i.e. the frequency response is forced to be flat at the 6-dB-down off-axis angles at all frequencies between the two critical frequencies.[1]

The corresponding high-pass crossover frequency response that drives the closer-spaced pair of sources is just simply the low-pass response subtracted from one, i.e.

$$H_{HP} = 1 - H_{LP} \qquad (6)$$

Note that both these low- and high-pass response functions are zero-phase.

The following graph (Fig. 9) shows several low-pass crossover frequency responses calculated using Eq. 5 for four example critical frequency or spacing ratios of 2.0, 2.5, 3.0, and 4.0. Note how the response curve drops gradually down to about -20 dB and then drops suddenly to infinite attenuation at the critical frequency. At all higher frequencies, the response is at maximum attenuation, or essentially off.

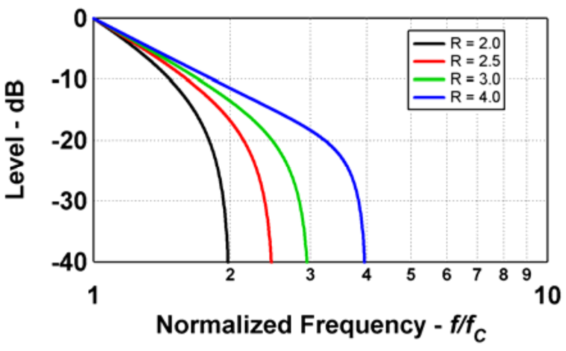


Fig. 9. Low-pass crossover responses for several critical frequency ratios: (left to right) R = 2.0 (black), 2.5 (red), 3.0 (green), and 4.0 (blue) calculated using Eq. 5.

The following graph shows several high-pass crossover frequency responses calculated using Eq. 6 for the same four example critical frequency or spacing ratios of 2.0, 2.5, 3.0, and 4.0 of the previous graph. All the responses rise suddenly from infinite attenuation at a normalized frequency of 1 and then gradually rise above a frequency of about 1.3 until the upper critical frequency where the response is at 0 dB or full on.

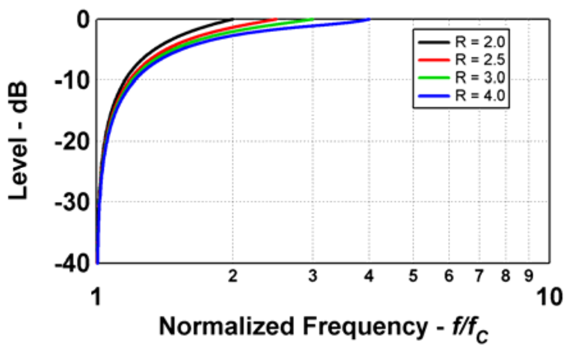


Fig. 10. High-pass crossover responses for several critical frequency ratios: (left to right) R = 2.0, 2.5, 3.0, and 4.0, calculated using Eq. 6.

The next graph (Fig. 11) shows individual plots of the high- and low-pass frequency-response combinations for the four spacing ratios illustrated in the previous two graphs: Fig. 11a: R = 2.0, Fig. 11b: R = 2.5, Fig. 11c: R = 3.0, and Fig. 11d: R = 4.0. Note that all four response plots have roughly the same shape and response up to the crossover frequency in the range from 1.29 to 1.44. Above a frequency of about 1.6 the responses change dramatically however and extend to higher frequencies as R increases.

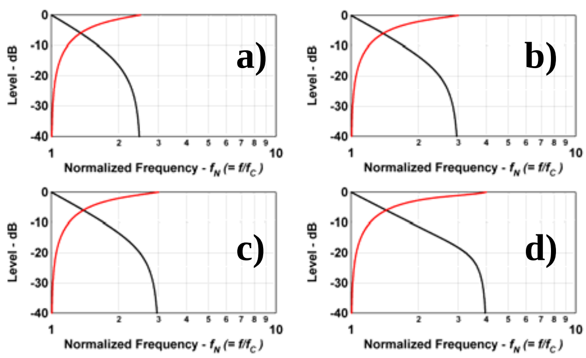


Fig. 11. Low-pass (black) and high-pass (red) crossover responses for a critical frequency ratios of a) 2, b) 2.5, c) 3.0, and d) 4.0.

2.5.2 *Response Between a Pair of Sources and Single Source*

Constant beamwidth can only be maintained over a limited range between a pair of sources and a single centrally-located source (the tweeter in this situation). To calculate the low-pass crossover function for the pair of sources, the following configuration is analyzed (Fig. 12).

[1] This is a special case of part 1 where the off-axis frequency response was defined to be flat at an arbitrary off-axis angle. Restricting the derivation to maintain constant 6-dB-down beamwidth simplifies the frequency response equations and the design procedure.

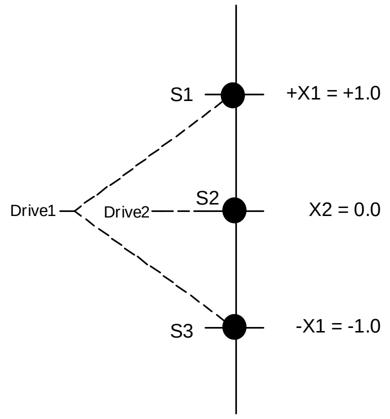


Fig. 12. Configuration of three point sources (S_1 – S_3) oriented vertically at normalized locations of -1.0, 0.0, and +1.0. The outside pair is driven by Drive1 (low pass) and the single inside source is driven by Drive2 (high pass). Note that the source strength of the center point source is twice that of the adjacent sources thus making their farfield pressure contributions equal.

To maintain constant beamwidth, the following equation (Eq. 7, again presented without proof or derivation) gives the low-pass crossover frequency response that drives the flanking pair of sources. As before, the high-pass response that drives the center source is just the low-pass response subtracted from one (see Eq. 6). This equation assumes that the source strength of the single central source is twice that of each of the flanking sources.

$$H_{LP}(f_N) = \begin{cases} \dfrac{1}{2}\left(\dfrac{1}{1-\cos(\frac{\pi f_N}{3})}\right) & \text{for } 1 \le f_N \le 3 \\ 0 & \text{otherwise} \end{cases} \qquad (7)$$

The equation is frequency normalized and covers only the range between the critical frequency of the outside pair and three times higher, i.e. $1 \le f_N \le 3$ where $f_N = f / f_{C1}$ and f_{C1} is the critical frequency for the outside pair where the spacing is a specific wavelength.

Note that it is impossible to extend the constant-beamwidth frequency range above three times the critical frequency of the outside pairs because the resultant polar does not drop below 6 dB from the on-axis level. Basically this means that at higher frequencies, the omni-directional polar pattern of the central point source dominates the polar pattern and the beamwidth can't be narrowed significantly by adding in a small amount of the output of the flanking pair of sources. Note also that the response function is somewhat inflexible in that it contains no parameter that allows adjustment of the crossover span such as provided by the parameter *R* in the previous equation. The crossover span is fixed at 3 to1.

The following graph shows the crossover responses calculated from Eqs. 7 and 6. Note that the attenuation of the low-pass response does not drop below -12 dB (x 0.25) at the highest frequency (f_N = 3). The corresponding high-pass response therefore does not rise above -2.5 dB (x 0.75) at the same frequency. Note that the sudden transition in the response at f_N = 3 can potentially cause filter implementation problems and can be smoothed by extending the response higher in frequency. Fig. 13 shows one such example extension (dashed line) which extends the response an octave higher based on linearly extending the responses. Note that the extension does not appear linear when plotted on the level-in-dB versus log-frequency scales as shown in Fig. 13.

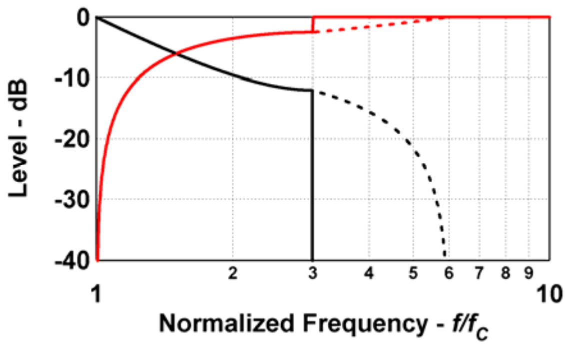


Fig. 13. Low-pass (black) and high-pass (red) crossover frequency responses calculated from Eqs. 6 and 7 that maintain constant beamwidth for the configuration of single point source flanked by two symmetrically located point sources shown in Fig. 12. Constant beamwidth can only be maintained up to a frequency three times higher than the critical frequency of the pair of flanking point sources. The dashed lines represent one possible extension of the responses that serves to smooth the sudden transition that occurs at this frequency to simplify filter implementation.

The following figure (Fig. 14) shows a polar overlay for the three-source configuration of Fig. 12 using the crossover function that maintains constant beamwidth (Eqs. 6 and 7). The figure plots six polars at one-third-octave normalized frequencies in the range of 1 to 3. A critical spacing of 0.5 wavelength is assumed for the two outside sources which provides a 83.6° beamwidth.

Note that all the polars coincide at the 6-dB-down beamwidth angle (indicated with short radial lines). Although the polars are very consistent within the rated (83.6°) beamwidth, the polar widens considerable at angles outside this angle range at the higher frequencies.

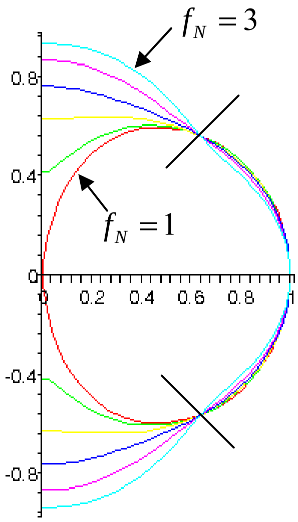


Fig. 14. Polar pattern overlays for the three-source configuration of Fig. 12, with a critical spacing of the outside sources of one-half wavelength, using the constant-beamwidth functions of Eqs. 6 and 7. The frequency span covers a three to one range starting at the critical frequency of the outside pairs and then rises at one-third-octave intervals covering 1, 1.25, 1.6, 2.0, 2.5 and 3.0. Note that all these polar curves exhibit constant beamwidth and coincide at the 6-dB-down points from on axis (noted on the graph with short black lines). The polar pattern widens at angles beyond the rated beamwidth as frequency increases. Note: all the polars are plotted with -40 dB at the center and 0 dB on the outside edge (same as previous polars but plotted using Matlab).

3 SUMMARY OF ARRAY DESIGN PROCEDURE

The following outline summarizes the design procedure for multi-way constant-beamwidth loudspeaker arrays.

1. Choose system specifications:
- a. Desired vertical beamwidth
- b. Operating range
- c. Height
- d. Driver complement

2. Calculate system parameters:
- a. Source separation-to-wavelength ratio (critical wavelength)
- b. Driver locations
- c. Driver critical frequencies
- d. Crossover frequency responses

These steps are described in the following.

3.1 Choose system specifications:

3.1.1 Desired vertical beamwidth:

The chosen beamwidth (-6 dB) should be in the range of about 65º to 120º. Beamwidths greater than 83.6º provide a wide lobe-free polar pattern with but with minimal far off-axis attenuation (see Figs. 6 and 7). A beamwidth of 83.6º provides maximum far off-axis attenuation with no side lobes (see Fig. 2c.) Beamwidths less than 83.6º exhibit narrower main beams, but with a single far off-axis lobe (see Fig. 2 d-f). See Figs. 3 and 4 for the tradeoffs of beamwidth and side lobe level versus source separation distance.

3.1.2 Operating range:

The constant-beamwidth operating range is governed by the number of driver pairs in the design and the chosen pair-to-pair step ratio. For example, a three-way design based on five drivers (a central tweeter with two pairs of flanking drivers) and a step ratio of three can provide at most a constant-beamwidth operating frequency range of somewhat greater than three octaves ($3^2 = 9$).

Correspondingly, a two decade operating range (say 100 Hz to 10 kHz) would require at least a five-way design ($3^4 = 81 \approx 100$). If a smaller step ratio of two was chosen, these systems would require a four-way system ($2^3 = 8 \approx 9$) and a seven-way system ($2^6 = 64 \approx 100$) respectively.

3.1.3 Height:

The height of the system governs how low in frequency the beamwidth can be maintained. The taller the system, the lower in frequency vertical beamwidth can be maintained. Roughly speaking, constant beamwidth is maintained down to the frequency where the height of the cabinet is one-half wavelength, i.e. $f_L = 0.5c/H$ where H = the cabinet height and c is the speed of sound.

3.1.4 Driver complement:

The drivers chosen for the system must have operating bandwidths that span each of their respective crossover bands. The response of the drivers in their bands must be essentially omni-directional and have flat frequency response or be equalized flat.

3.2 Calculate system parameters:

3.2.1 Source separation-to-wavelength ratio (critical wavelength):

Once the beamwidth has been chosen, the source separation-to-wavelength ratio for all the pairs of sources (loudspeakers) must be determined from the graph in Fig. 3 or using Eq. 3. Note that this is a single ratio that applies to all the pairs of sources that make up the array. For example, if a beamwidth of 83.6º is chosen, this corresponds to a separation-to-wavelength ratio of 0.5 i.e., each pair of speakers that make up the array are spaced one-half wavelength apart at each of their respective critical frequencies.

3.2.2 Driver locations:

The drivers must be located on the front panel in pair-wise symmetrical combinations starting from the center tweeter and moving outward towards the top and bottom of the array. In this process, the pair-to-pair spacing ratio must be determined. Note that unlike the source separation-to-wavelength ratio which is a constant over all the driver pairs, the spacing step ratio and the corresponding critical frequency ratios can vary from source pair to source pair. This ratio must typically be limited to the range of about 2:1 to 3:1 and preferably no higher than 2.5:1.

Typically the pair of speakers that flank the tweeter are spaced as close to the tweeter as possible. This allows this pair of sources to operate up to the highest possible frequency to control beamwidth. Remember that as soon as the tweeter starts operating alone, the vertical coverage of the system is only limited by the inherent high-frequency coverage of the tweeter itself.

Once the tweeter and its two flanking pair of sources are located, the remaining pairs of drivers should be located. Typically a larger spacing step ratio is chosen to maximize the spacing of the remaining sources. This maximizes the operating range of the speaker. The spacing must still be limited so that the spacing of the lowest frequency sources do not exceed the height of the cabinet.

3.2.3 Driver critical frequencies:

Once the drivers have been located and the spacing step ratios have been set, the crossover critical frequencies must be chosen for each pair of speakers. Note that the spacing step ratios and ratios between each pair's critical frequencies are equal. The critical frequencies themselves are then calculated from Eq. 1 using the actual locations of the drivers. Note that all dimensions must be referenced to the location of the tweeter, which is assumed to be located at the center of the coordinate system.

3.2.4 Crossover frequency responses:

After the crossover critical frequencies have been calculated, the actual crossover frequency responses can be calculated using Eqs. 5 - 7. Note that the in addition to knowing the critical frequencies of each pair of sources, the frequency step ratios from each pair to the next pair must also be known to calculate the crossover responses.

The frequency responses of the individual drivers can be incorporated in the crossover by including the measured complex frequency response of the drivers in the calculation of the crossover filters responses. Part 1 implements the crossover filters with finite impulse response (FIR) DSP filters. Note that required driver equalization may be included in the FIR filters.

The last crossover transition between the pairs of sources that flank the tweeter and the tweeter itself must be handled separately using Eqs. 7 and 6. At the highest frequencies (above 3 times the critical frequency of the flanking pair), the tweeter is operating alone and thus may provide a wide uncontrolled polar response. This uncontrolled widening can be compensated for somewhat by the use of crossover optimization and the use of a tweeter whose polar response is more controlled.

4 EXAMPLE ARRAY DESIGN

This section illustrates the design and simulation of an example five-way constant-beamwidth loudspeaker array. For design purposes, it is assumed that the maximum external dimensions of the drivers are equal to their stated driver size.

4.1 Desired system specifications:

a. Vertical beamwidth of about 75º with side lobes down at least 17 dB.

b. Constant-beamwidth operating range of about 100 Hz on up.

c. Height = 2 m (6.7 ft, 80 in) approximately.

d. Use two 15” sub woofers, two 8” woofers, two 4” lower midranges, two 2” upper midranges, and a single 1” dome tweeter.

4.2 System Parameters and Implementation:

4.2.1 Source separation-to-wavelength ratio (critical wavelength):

The chosen 75º vertical beamwidth and the lobe requirement dictates a critical driver spacing of about 0.55 wavelength (from Fig. 3 or Eq. 2).

4.2.2 Driver locations:

The layout should start from the tweeter at the center of the array and work outward. To maximize the high-frequency constant-beamwidth operating range, the first pair of drivers flanking the tweeter should be located as close together as possible. This dictates that the two 2” upper midranges should be mounted as close to the tweeter as possible.

The remaining drivers should be located above and below the tweeter and distributed so that the 15” woofers are near the top and bottom of the cabinet. Preferably, the spacing step ratio should held to no more than 3:1 and preferable less than 2.5:1.

Figure 15 shows the final front panel layout and driver locations. Although the two upper midranges are mounted as close to the tweeter as possible with their centers only 1.5” above and below the tweeter, the center-to-center spacing is still a relatively large at 3”.

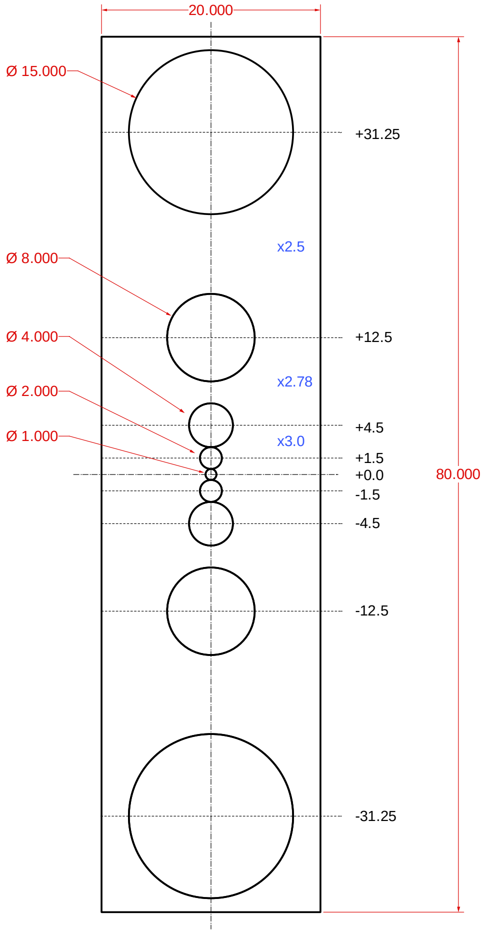


Fig. 15. Driver locations and front panel layout for the example five-way constant-beamwidth loudspeaker system. The system uses two 15" sub woofers, two 8" woofers, two 4" lower midranges, two 2" upper midranges, and a single 1" dome tweeter. The driver spacing ratios are indicated with "x" prefixes. All dimensions in inches.

4.2.3 Driver critical frequencies:

The critical frequency of each pair of drivers is calculated next. This is calculated from the center-to-center spacing of the drivers using Eq. 1 (assuming a 0.55-wavelength spacing, this equation appears as: $f_C = 0.55c/D = 7{,}425/D$, where D = center-to-center driver spacing in inches). Table 2 shows the system parameters for this design.

Table 2: System Parameters

Driver	Location (Inches)	Pair Spacing (Inches)	Critical Freq. (Hz)	Step Ratio	Crossover Frequency (Hz)
15" Sub Woofer	±31.25	62.5	119	2.5	160
8" Woofers	±12.50	25.0	297	2.78	408
4" Lower Midranges	±4.50	9.0	825	3.00	1,150
2" Upper Midranges	±1.50	3.0	2,475	-	3,372
1" Tweeter	0.00	-	-	-	-

4.2.4 Crossover frequency responses:

With the exception of the highest crossover to the tweeter, the crossover frequency responses were calculated using Eqs. 5 and 6. The crossover between the upper midranges and the single tweeter was calculated using Eqs. 7 and 6 with no extension added to smooth the sudden transition which occurs at 7.4 kHz in this design.

Note that the critical frequency of the upper midranges is a fairly-low 2.5 kHz. This means that the whole upper range of the system above this frequency will be carried by the upper midranges and the single tweeter. Note that the driver configuration can only maintain constant beamwidth up to a frequency three times the highest critical frequency, which in this design is about 7.4 kHz (= 3 x 2,475 Hz).

Assuming an omni-directional tweeter, the system's 75º beamwidth will only be maintained up to 7.4 kHz where it will suddenly increase to 180º and then remain at this value up to 20 kHz. As noted before, constant beamwidth can only be maintained in this highest frequency range by the use of crossover optimization techniques coupled with the use of a tweeter that has a narrower more-controlled coverage at high frequencies.

Fig. 16 shows the crossover frequency responses calculated for this array using Eqs. 5, 6, and 7.

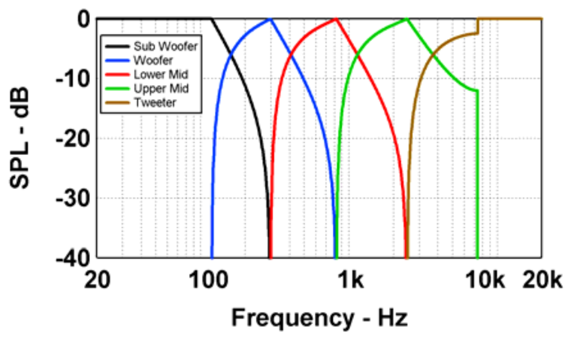


Fig. 16. Crossover frequency responses for the example five-way constant-beamwidth speaker system shown in Fig. 15. The response exhibits critical frequencies (pointed tops) at 119 Hz, 297 Hz, 825 Hz, and 2.475 kHz; and crossover frequencies (-6 dB) of 160 Hz, 408 Hz, 1.15 kHz, and 3.72 kHz. Above 7.4 kHz, the tweeter operates on its own with full drive. Below 119 Hz, the subwoofers operate alone at full drive.

The following three figures (Figs. 17-19) show the results of performance simulations of the example array. Far field performance is simulated. Performance characteristics including beamwidth (Fig. 17) and directivity (Fig. 18) versus frequency, and vertical-plane polar plots at one-third-octave intervals from 20 Hz to 20 kHz (Fig. 19). The horizontal-plane polar responses are not shown because all the polars are omni-directional.

Note the extreme uniformity of beamwidth, directivity, and main lobe polar shape in the frequency range of 125 Hz to 5 kHz. Note the nearly identical polar shapes that occur at the one-third-octave center frequencies closest to the arrays critical frequencies (125, 315, 800, and 2500 Hz). Between these frequencies however, the polar shapes exhibit greater variability, particularly for far off-axis angles.

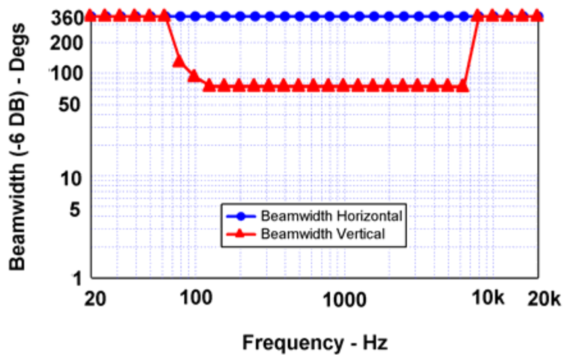


Fig. 17. Beamwidth (-6 dB) versus frequency of the example five-way constant-beamwidth speaker system shown in Fig. 15. Note the extreme uniformity of beamwidth which is about 75º between 125 Hz and 6.3 kHz. At 7.4 kHz and above, the tweeter (assumed to be a point source) is radiating by itself and provides an omni-directional 180º response. This can be corrected by using a tweeter with an inherent controlled high-frequency coverage.

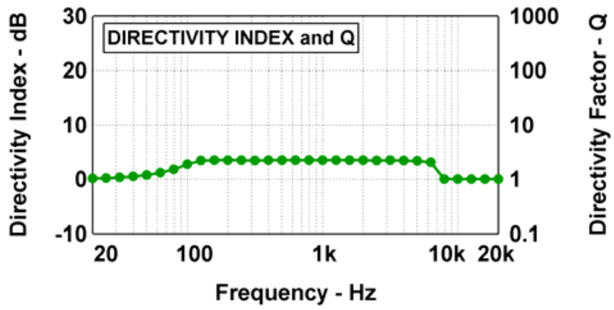


Fig. 18. Directivity versus frequency of the example five-way constant-beamwidth speaker system shown in Fig. 15. Note the extremely uniform +3.5 dB directivity index between 125 Hz and 6.3 kHz. The directivity drops above 6.3 kHz due to the omni-directional tweeter which is operating alone. The pass-band directivity can potentially be 3 dB higher depending on how the rear radiation is attenuated.

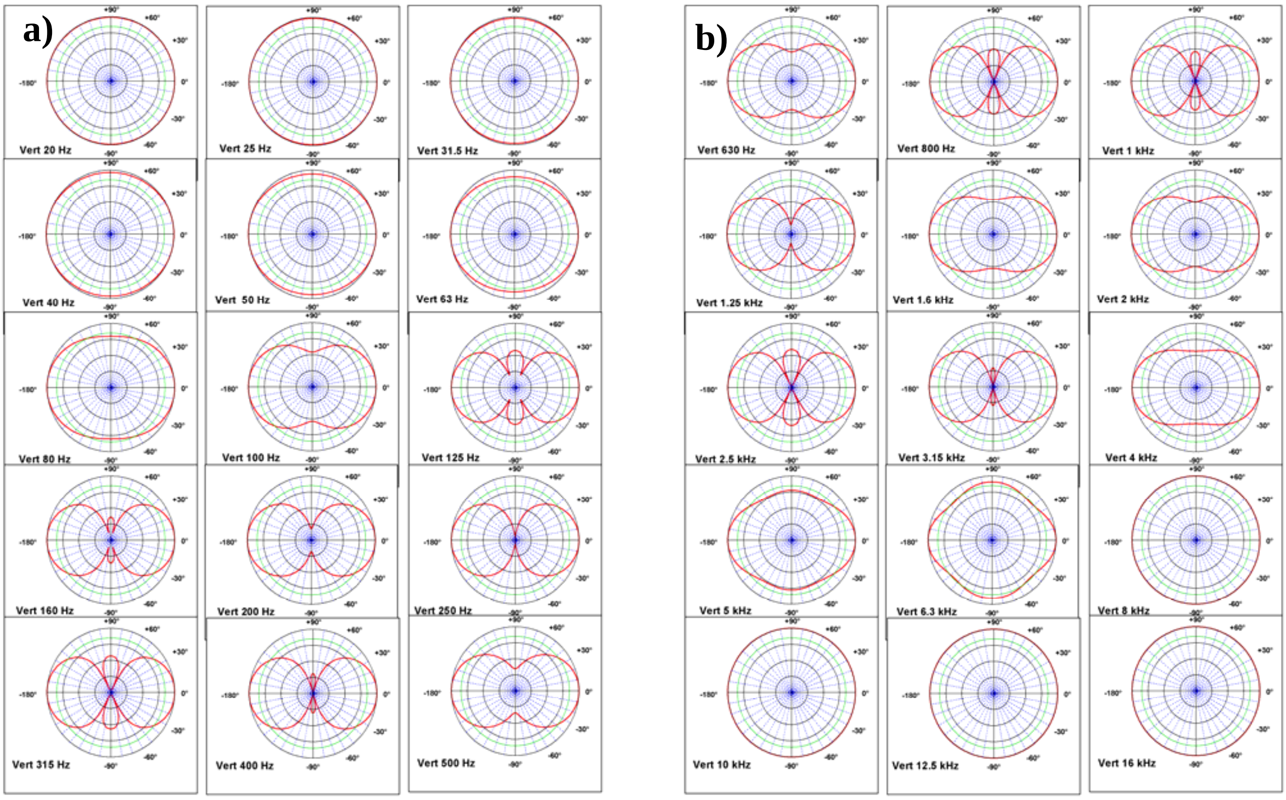


Fig. 19. Vertical-plane polar patterns for the example five-way constant-beamwidth speaker system shown in Fig. 15 at one-third-octave frequencies. (a) 20 to 500 Hz. (b) 630 Hz to 16 kHz (left to right and top to bottom in each grouping). All the polar patterns are front-rear symmetric. Unlike Figs. 6-8, the rear responses are also shown. Note the polar responses at 125 Hz, 315 Hz, 800 Hz and 2.5 kHz, which are near the critical frequencies of the design. In between these frequencies the polar shapes vary somewhat but maintain a very constant frontal shape with unvarying coverage angle. Below 100 Hz and above 8 kHz, the polar shapes are omni-directional because the woofers and tweeter are operating by themselves

5 SUMMARY

In this paper we described a new linear-phase DSP technique for crossing over multi-way loudspeakers utilizing pair-wise symmetric driver configurations with a central tweeter in a vertical array. The technique is based on combining the acoustic outputs of pairs of drivers to yield a flat frequency response at an arbitrary specified off-axis angle. When thus flattened, responses at other off-axis angles are found to be fairly flat as well.

In contrast to prior crossover techniques such as Linkwitz-Riley, constant-voltage, high-order notched, etc., the new technique actually maintains flat off-axis frequency response throughout most of the operating range of the speaker except at high frequencies where the single central tweeter operates on its own.

The technique produces a crossover filter frequency response with a very distinctive pointed-top shape. On either side of the point, called a critical frequency, the response rolls off rapidly and essentially shuts off at frequencies above and below the critical frequencies of the adjacent drivers. At a critical frequency, only one pair of drivers are energized. At frequencies between the critical frequencies, only two pairs of speakers are operating.

In Part 2 of this paper (this part), we described a some-what simplified version of the design technique described in Part 1 that places emphasis on maintenance of constant beamwidth, directivity, and uniformity of polar shape with frequency. The simplification was accomplished by restricting the flattened off-axis frequency response to the 6-dB-down level from on axis. Thus restricted, the 6-dB level now corresponds to the level at which beamwidth is defined. Forcing the 6-dB-down off-axis frequency response to be flat, also forces the beamwidth to be constant in the same frequency region.

6 ACKNOWLEDGEMENTS

This work was supported by Harman International, Harman Consumer Group, and Harman/Becker Automotive Systems.

All calculations and graphing in Part 1 were accomplished using MatLab (www.mathworks.com/) and in Part 2 using Igor Pro (www.wavemetrics.com/).

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